MATH 165, Final

1. (4 points) Prove the following statement using the ϵ–δ definition of limit.

$$\lim_{x\to 2}(3x+4) = 10.$$

2. (20 points) Decide if the following limits exist. When they do, compute them.

a) $\lim_{x \to 2} \frac{x^2+4}{x^2-x-2}$

b) $\lim_{x \to 2} \frac{x^2+4}{x^2+x+2}$

c) $\lim_{x \to 2} \frac{-x^2-2x+8}{x^2-x-2}$

d) $\lim_{x \to -\infty} \frac{2x+3}{\sqrt{x^2+2}}$

3. (15 points) Compute the derivatives of the following functions:

(a) $f(x) = \dfrac{2}{x^3} - \sqrt{x^3 - 4x + 7}$

(b) $g(x) = \sin(\tan x^2)$

(c) $h(x) = \dfrac{2x^2 + 1}{3x - 1}$

4. (5 points) Use implicit differentiation to find the equation of the tangent line to the given curve at $(1, 1)$.

$$x^2 + 2xy + y^2 = 4$$

5. (9 points) Let $f(x) = x^4 - 2x^2 - 2$.

(a) Where is f increasing?

(b) Where is f concave up?

(c) What are the local maxima and minima?

6. (5 points) Find the horizontal and vertical asymptotes of the curve $y = \frac{2x^2-5}{x^2+x-2}$. Make sure that your answers are supported by the appropriate limits.

7. (5 points) What is the smallest slope among all tangent lines of the graph of $y = 2x^3 - 2x^2 + x + 1$?

8. (4 points) Set up, but **do not calculate** the Riemann Sum for the area under the graph of the function $f(x) = 2 + x^3$ for $2 \leq x \leq 5$. Use RIGHT-HAND end points and n subintervals.

9. (20 points) Evaluate

a) $\displaystyle\int_0^1 \frac{x}{\sqrt{3x^2+1}}\,dx$

b) $\displaystyle\int_{-1}^1 \frac{x}{\sqrt{x^4+5}}\,dx$

c) $\displaystyle\int \cos x\,\sin^5 x\,dx$

d) $\displaystyle\int x\sec(x^2)\tan(x^2)\,dx$

10. (5 points) Sketch the region enclosed by the given curves in the first quadrant and a typical approximating rectangle. Then find the area of the region. $x = y$ and $x = y^3$.

11. (12 points) Let A be the region bounded by the graph of $y = \tan x$, the x-axis and the line $x = \pi/4$. Set up, but **do not calculate** the integrals to find the volume of the solid generated by revolving it around

a) the y-axis   b) the x-axis   c) the line $x = 2$.

Bonus. (5 points) Find a positive continuous function f such that the area under its graph on the interval [0,x] is x^5.