

NO CALCULATORS ALLOWED

December 8, 2007

MATH 163 FINAL EXAM (100 points)

Fall 2007

SHOW YOUR WORK, PLEASE

Problem 1 (5 points) Given that $\lim_{x \rightarrow 5} \frac{x}{4} - 1 = \frac{1}{4}$, illustrate the **Precise Definition** of a limit (in other words the $\epsilon - \delta$ Definition of a limit) by finding a value of δ that corresponds to $\epsilon = \frac{1}{8}$.

Problem 2 (15 points) Find the limits, if they exist.

a. $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1}$

b. $\lim_{x \rightarrow -1} \frac{x^2 + 1}{x - 1}$

c. $\lim_{x \rightarrow \infty} \frac{x^2 - 1}{3x^2 + 1}$

Problem 3 (10 points) Compute the derivative of f

a. $f(x) = \frac{1}{x^5} - \sqrt[5]{3x^3 - x + 5}$

b. $f(x) = \tan^2 x - \tan x^2$

Problem 4 (5 points) Find an equation to the tangent line to the graph of $f(x) = \sqrt{x^2 - 1}$ at the point $(2, \sqrt{3})$.

Problem 5 (10 points) (a). Find the intervals on which the function $f(x) = 2 + 3x - x^3$ is increasing and decreasing.

(b) Where is f concave up?

Problem 6 (10 points) Sketch the graph of the function $f(x) = \frac{2x^2}{9 - x^2}$. This function has the following derivatives: $f'(x) = \frac{36x}{(9 - x^2)^2}$ and $f''(x) = \frac{108(x^2 + 3)}{(9 - x^2)^3}$. DO NOT recalculate the derivatives; just USE them!

Problem 7 (10 points) A builder intends to construct a storage shed having a volume of 900 ft³, a flat roof, and a rectangular base whose width is three-fourths the length. The cost per square foot of the material is \$4 for the floor, \$6 for the sides and \$3 for the roof. What dimensions will minimize the cost?

Problem 8 (5 points) Evaluate the integral by interpreting it in terms of an area

$$\int_0^2 \sqrt{(4-x^2)} dx$$

Problem 9 (15 points) Evaluate

a. $\int_1^2 \left(\frac{3}{x^2} - \sqrt{2x+3}\right) dx$

b. $\int (\sin x \cos x + \tan x \sec x) dx$

c. $\int (x \sin x^2 + \sec^2 x) dx$

Problem 10 (5 points) Sketch and find the area of the region bounded by the graphs of $y = x$, and $y = 2 - x^2$.

Problem 11 (10 points) Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = x$.

(a) Find the volume of the solid generated by revolving R about the x -axis.

(b) Find the volume of the solid generated by revolving R about the y -axis.

Bonus Problem (5 points) Calculate the given limit

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i}{n^2}$$