

Math 163 Final Exam, Spring 2008

1. (10 points) Find the limits if they exist.

(a)

$$\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x - 2}$$

(b)

$$\lim_{x \rightarrow \infty} \frac{2x^2 - 4x + 1}{x^2 + 5}$$

2. (5 points) Given

$$\lim_{x \rightarrow 1} (3x - 1) = 2,$$

illustrate the $\epsilon - \delta$ definition of the limit by finding δ when $\epsilon = 1/6$.

3. (10 points) Calculate the derivative of f .

(a)

$$f(x) = \frac{x^2 + 1}{\sqrt{2x + 3}}$$

(b)

$$f(x) = \sin(3x^2 + 2x - 4)$$

4. (8 points) Find the equation of the tangent line to the graph of $f(x) = (2x - 1)^3$ at the point $(2, 27)$.

5. (8 points) The base of a rectangle is increasing at the rate of 3 inches/minute and the height is decreasing at the rate of 2 inches/minute. How fast is the area changing when the length of the base is 4 feet and the height is 2 feet?

6. (12 points) Given the function $f(x) = x^3 - 6x^2 + 9x + 1$.

- (a) Find the critical points of f .
- (b) Find the intervals where f is increasing and decreasing.
- (c) Find the local maximum and minimum of f .
- (d) Find the intervals where f is concave upwards and concave downwards.
- (e) Find the inflection points of f .
- (f) Sketch the graph of f .

7. (8 points) Show that of all the right triangles with hypotenuse of length 10, a $(45^\circ - 45^\circ - 90^\circ)$ triangle has the greatest area.

8. (10 points) Evaluate the integrals

(a)

$$\int_0^{\pi/2} \sin^3 x \cos x \, dx$$

(b)

$$\int x(5x^2 - 3)^4 dx$$

9. (5 points) Compute the derivative of

$$f(x) = \int_1^x (t^3 - 1)^4 dt$$

10. (8 points) Sketch and find the area between the graphs of $f(x) = 6 - x^2$ and $g(x) = -x + 4$.

11. (8 points) Set up but **DO NOT EVALUATE** the integral for the volume of the solid obtained by rotating the region bounded by the x -axis, the lines $y = 1$ and $y = 4$ and the graph of $y = 1/\sqrt{x}$ around the x -axis.

12. (8 points) The integral

$$\int_0^1 \pi(1-y)^2 dy$$

represents the volume of a solid. Describe and/or sketch (with labels) such a solid.

Bonus. (8 points) Evaluate the integral

$$\lim_{h \rightarrow 0} \frac{\int_4^{4+h} \frac{1}{t} dt}{h}$$