Math 165 Final Exam, Fall 2008

1. A positive number ε and the limit L of a function f at a are given. Find a number δ such that $|f(x) - L| < \varepsilon$ if $|x - a| < \delta$

$$\lim_{x \to 3}(5x - 2) = 13; \ \varepsilon = 0.01.$$

Justify your answer.

2. Find the limit, if it exists.

(a)

$$\lim_{x \to 2} \frac{x^2 - 4}{x + 2}$$

(b)

$$\lim_{x \to 2} \frac{x^2 + 2x - 8}{x^2 - x - 2}$$

(c)

$$\lim_{x \to \infty} \frac{4x^2 - 3}{x^2 - x + 2}$$

3. Compute the derivative of f.

(a)

$$f(x) = x^{-5} - \sqrt[3]{x^3 - 4x + 7}$$

(b)

$$f(x) = \frac{x+1}{\sin 5x}$$

4. Find the point where the tangent line to the graph of $f(x) = 2\sin x - x$ is horizontal for $0 \leq x \leq \pi/2$.

5. If $f(x) = x + \frac{1}{x}$ then $f'(x) = 1 - \frac{1}{x^2}$ and $f''(x) = \frac{2}{x^3}$.

   (a) Where is f increasing?

(b) Where is f concave up?

(c) What are the local maximums and minimums?

6. Find the inflection point(s) of the graph of the function $y = x^4 - 2x^3 + 3x + 1$.

7. Find the point on the parabola $x + y^2 = 0$ that is closest to the point (0,-3).

8. Set up, but **do not calculate** the Riemann Sum for the area under the graph of the function $f(x) = 2 + x^3$ for $2 \leq x \leq 5$. Use RIGHT-HAND end points and n subintervals.

9. Evaluate

(a)

$$\int_0^3 \frac{1}{\sqrt{x+1}}\,dx$$

(b)

$$\int_{-3}^{3} \frac{x}{\sqrt{x^4+1}}\, dx$$

(c)

$$\int \cos 2x \, \sin^2 2x \, dx$$

(d)

$$\int x \sec^2(x^2)\, dx$$

10. Sketch and find the area of the region bounded by the graphs of $y = x$, and $y = x^3 - 3x$.

11. Find the volume of the solid generated by rotating the region bounded by the graphs of $y = x^{2/3}$ and $y = x$ about the x-axis.

12. Let A be the region bounded by the graph of $y = \sin x$, and the x-axis for $0 \leq x \leq \pi$. Set up, but **do not calculate** the integrals to find the volumes of the solids generated by revolving it around

(a) the y-axis

(b) the x-axis

(c) the line $y = -1$.

Bonus Let

$$f(x) = \int_0^x \sqrt[3]{\cos x}\; dx.$$

Find the critical points in the interval $[0, 2\pi]$.