Math 165 **Final Exam** **May 2, 2009**

Exam is 8 pages plus cover page. Follow the instructions for each question.
Show enough of your work that we can understand what you are doing.

1. Find the limit, or, if it does not exist, say so.

(4 points) (a) $\displaystyle\lim_{x\to -1} \frac{2x^2+x-1}{5x^2+6x+1}$

(4 points) (b) $\displaystyle\lim_{x\to \infty} \frac{2x^2+x-1}{5x^2+6x+1}$

(4 points) (c) $\displaystyle\lim_{x\to 2} \frac{x^2+x-1}{3x^2+2x+1}$

(8 points) **2.** Using the definition of the derivative as a limit of difference quotients, find $p'(3)$ for $p(x) = 2x^2 - 4x + 7$. Begin your work by stating the definition of $p'(3)$ as a limit.

(8 points) **3.** Find an equation for the line tangent to the parabola $y^2 - 3y + 2x = 6$ at the point $(1, -1)$.

4. Find the derivatives of the following functions.

(5 points) (a) $f(x) = \dfrac{x^3}{5} + \dfrac{3}{x^2} + \sqrt{2\pi}$

(5 points) (b) $g(\theta) = \theta^2 \tan 5\theta$

(5 points) (c) $h(r) = \dfrac{\sqrt{r^2 + 4r}}{3 - 2r}$

(8 points) **5.** Satellite measurements of a large rectangular block (of an unknown material) show that it is changing in size. These measurements show that the length of the block is decreasing at the rate of 2 feet per hour, the width of the block is decreasing at the rate of 1 foot per hour and the height of the block is increasing at the rate of 1/2 foot per hour. At the moment that the block is 120 feet long, 50 feet wide, and 10 feet high, is the volume of the pile increasing or decreasing? At what rate is the volume increasing or decreasing? (Be sure to give units for your answer!)

(8 points) **6.** Find the maximum value and the minimum value of the polynomial $p(x) = x^3 - 5x^2 + 3x + 6$ on the interval $-2 \leq x \leq 4$.

7. Let f be the function $f(x) = 3x^4 + 2x^3 - 3x^2 - 2$

(2 points) (a) On which intervals is f increasing? ______________________________

(2 points) (b) On which intervals is f decreasing? ______________________________

(2 points) (c) For which value(s) of x does f have a local maximum?____________________

(2 points) (d) For which value(s) of x does f have a local minimum?____________________

8. Evaluate the following integrals:

(5 points) (a) $\int_0^3 x^2 - 5\sqrt{x+1} + 4x\,dx$

(5 points) (b) $\int \frac{3r}{(1-r^2)^3}\,dr$

(5 points) (c) $\int 3\sin 2x \cos 2x + 4\sec 5x \tan 5x\,dx$

(2 points) **9.**(a) Sketch the graphs of the equations $x - y + 1 = 0$ and $y - x^2 + 1 = 0$ and then shade the part of the region between them that satisfies $x \geq 0$.

(8 points) (b) **Set up** an integral (or integrals) to find the volume of the solid obtained by rotating around the y-axis the region between the graphs of $x - y + 1 = 0$ and $y - x^2 + 1 = 0$ with $x \geq 0$ (the region above). (Do *not* evaluate the integral(s).)

(2 points) (c) On your sketch above, indicate the typical rectangle(s) that correspond to your choice of how to set up the integral(s).

(8 points) **10.** Find the area, as *a number*, between the curves $x = 2y - y^2$ and $y = x + 2$.

Bonus:

(8 points) **Set up** (but do not evaluate) a Riemann sum with n subintervals, using the **right end points** of each subinterval that would be suitable for use in finding the integral

$$\int_1^3 x^3 + 2x\,dx$$