IUPUI Department of Mathematical Sciences
Departmental Final Examination

PRACTICE FINAL EXAM VERSION #2

MATH 15300

College Algebra

Exam directions similar to those on the departmental final.

1. DO NOT OPEN this test booklet until you are told to do so.
2. This is NOT the exam for MATH 15400 or 15900.
3. There are 7 pages in this exam with problems 1 to 24 and a bonus problem.
4. You MUST get a new exam from the proctor if your exam is incomplete.
5. PRINT your name and student ID# below.
6. MARK your section below.
7. You will have two hours to complete this examination.
8. A TI-30Xa calculator is permitted, no other calculator is allowed.
9. No scrap paper, notes, books, or collaborators are allowed.
10. Exact answers may contain π or radicals or logarithms.
11. Simplify all answers completely.
12. Problems involving units must have the units represented on the answer to receive full credit.

Name (Print Clearly)	Solutions
Student ID#	

Practice Departmental Final Exam Recommendations to Students:

- Take this practice final exam like an actual examination (not like doing homework). That is, create an "exam like" atmosphere. This practice exam should be taken after completing a thorough review of the material.
- Set aside a two-hour block of time with no interruptions (no facebook, texting, phone calls, restroom breaks, etc.).
- Do not use any help aids, such as notes, textbook, internet, scrap paper, MAC staff, etc.
- Work through all problems noting which concepts you know well and which ones you need to spend more time on.
- Grade your exam using the answers in the back of your textbook (the textbook section and exercise number is noted at the top right of each problem).
- Rework any problem on the exam that you missed and then work similar problems from the textbook until you can perform the operations without error.
- Follow the same recommendations for taking the Practice Final Exam Version #1.

MATH 15300 Departmental Practice Final Exam (Version #2)

TEXTBOOK: Swokowski & Cole, *Algebra & Trigonometry with Analytic Geometry*, Classic 12th Edition

To receive full credit you must show all your work. Simplify all answers completely. Be sure to check your final answers for errors. Problems involving units must have the units represented on the answer to receive full credit.

1. Simplify the expression, assuming x and y may be negative. (1.2 #83)

$$\sqrt[4]{x^8(y-1)^{12}} = |x^2||(y-1)^3|$$
$$= x^2(y-1)^2|(y-1)|$$

1. $= x^2(y-1)^2|(y-1)|$ (4)

2. Express as a polynomial. (1.3 #35)

$$\left(x^{1/3}-y^{1/3}\right)\left(x^{2/3}+x^{1/3}y^{1/3}+y^{2/3}\right)$$
$$= x + x^{2/3}y^{1/3} + x^{1/3}y^{2/3} - x^{2/3}y^{1/3} - x^{1/3}y^{2/3} - y$$
$$= x - y$$

2. $= x-y$ (4)

3. Simplify the expression. (1.4 #73)

$$(3x+1)^6\left(\tfrac{1}{2}\right)(2x-5)^{-1/2}(2) + (2x-5)^{1/2}(6)(3x+1)^5(3)$$
$$= (3x+1)^6(2x-5)^{-1/2} + 18(3x+1)^5(2x-5)^{1/2}$$
$$= (3x+1)^5(2x-5)^{-1/2}[(3x+1)+18(2x-5)]$$
$$= \frac{(3x+1)^5(39x-89)}{(2x-5)^{1/2}}$$

3. $= \dfrac{(3x+1)^5(39x-89)}{(2x-5)^{1/2}}$ (4)

4. Solve for the specified variable. (2.1 #41)

$$\frac{2}{2x+1} - \frac{3}{2x-1} = \frac{-2x+7}{4x^2-1}$$
$$2(2x-1) - 3(2x+1) = -2x+7$$
$$4x - 2 - 6x - 3 = -2x+7$$
$$-2x-5 = -2x+7$$
$0 = 12$, a contradiction, no solution

4. No solution (4)

5. ***Delivering Newspapers*** It takes a girl 45 minutes to deliver the newspapers on her route; however, if her brother helps, it takes them only 20 minutes. How long would it take her brother to deliver the newspapers by himself? (2.2 #33)

Let x = time for brother by himself

$$\frac{1}{45}+\frac{1}{x}=\frac{1}{20} \qquad x = 36 \text{ minutes}$$

$$4x+180=9x$$

$$5x=180$$

5. 36 minutes (4)

6. Solve for the specified variable. (2.3 #53)

$A = 2\pi r(r+h)$ for r

$$A = 2\pi r^2 + 2\pi hr$$

$$2\pi r^2 + 2\pi hr - A = 0$$

$$a = 2\pi,\ b = 2\pi h,\ c = -A$$

Use quadratic formula

$$r = \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 - 4(2\pi)(-A)}}{4\pi}$$

$$r = \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 8\pi A}}{4\pi}$$

$$r = \frac{-\pi h \pm \sqrt{\pi^2 h^2 + 2\pi A}}{2\pi}$$

since $r>0$ we only use + sign.

6. $r = \dfrac{-\pi h + \sqrt{\pi^2 h^2 + 2\pi A}}{2\pi}$ (4)

7. Find the solutions of the equation. (2.4 #56)

$8x^3 - 12x^2 + 2x - 3 = 0$

$$4x^2(2x-3)+1(2x-3)=0$$

$$(2x-3)(4x^2+1)=0$$

$$2x-3=0 \qquad 4x^2+1=0$$

$$x = 3/2 \qquad 4x^2=-1 \qquad x^2 = -\frac{1}{4} \qquad x = \pm\frac{1}{2}i$$

7. $x = \frac{3}{2},\ \pm\frac{1}{2}i$ (4)

8. Solve the equation. (2.5 #5)

$3|x+1| - 2 = -11$

$$3|x+1| = -9$$

$$|x+1| = -3$$

Absolute value is nonnegative

$|x+1|=-3$ has no solution

8. No solution (4)

9. Solve the inequality, and **express the solutions in terms of intervals** whenever possible. (2.6 #65)

$\dfrac{3}{|5-2x|} < 2$

$$|5-2x| > \frac{3}{2}$$

$$5-2x < -\frac{3}{2} \text{ or } 5-2x > \frac{3}{2}$$

$$-2x < -\frac{13}{2} \text{ or } -2x > -\frac{7}{2}$$

$$x > \frac{13}{4} \text{ or } x < \frac{7}{4}$$

7/4 13/4

9. $(-\infty, 7/4) \cup (13/4, \infty)$ (4)

10. Solve the inequality, and **express the solutions in terms of intervals** whenever possible. (2.7 #31)

$$\frac{x+1}{2x-3} > 2$$

zero: $x = 7/3$

Division by zero: $x = 3/2$

$$\frac{x+1}{2x-3} - 2 > 0$$

$$\frac{x+1-2(2x-3)}{2x-3} > 0$$

$$\frac{-3x+7}{2x-3} > 0$$

	$x < 3/2$	$3/2 < x < 7/3$	$x > 7/3$
$(-3x+7)$	+	+	−
$(2x-3)$	−	+	+
Quotient $\frac{-3x+7}{2x-3}$	−	+	−

$(3/2, 7/3)$

10. $(3/2, 7/3)$ (4)

11. Given the points $A(-4,-3)$ and $B(6,1)$. Find a formula that expresses the fact that an arbitrary point $P(x,y)$ is on the perpendicular bisector l of segment AB. (3.1 #23)

$d(P,A) = d(P,B)$

$$\sqrt{(x+4)^2+(y+3)^2} = \sqrt{(x-6)^2+(y-1)^2}$$

$$x^2+8x+16+y^2+6y+9 = x^2-12x+36+y^2-2y+1$$

$$20x+8y=12$$

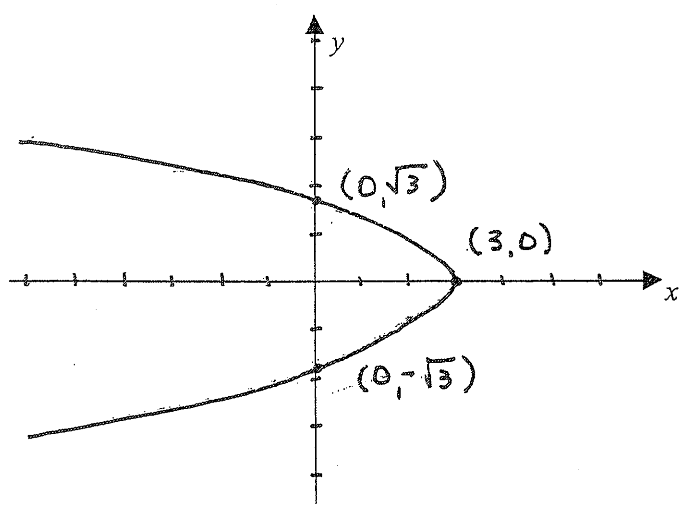
$$5x+2y=3$$

11. $5x+2y=3$ (4)

12. Sketch the graph $x = -y^2+3$, and label the x- and y-intercepts. (3.2 #11)

(4)

x	y
3	0
0	$\pm\sqrt{3}$



13. Find an equation of the circle with center $C(-4,6)$ and passing through the point $P(1,2)$. (3.2 #39)

$$(x-h)^2+(y-k)^2=r^2$$

$$(1+4)^2+(2-6)^2=r^2$$

$$25+16=r^2$$

$r^2=41$ $\quad (x+4)^2+(y-6)^2=41$

13. $(x+4)^2+(y-6)^2=41$ (4)

14. Find a general form of an equation of the line through the point $A(7,-3)$ that is perpendicular to the line $2x-5y=8$. (3.3 #31)

$-5y=-2x+8$

$y=\frac{2}{5}x-\frac{8}{5}$

$m_{\perp}=-\frac{5}{2}$

$y+3=-\frac{5}{2}(x-7)$

$2y+6=-5x+35$

$5x+2y=29$

14. $5x+2y=29$ (4)

15. Given $f(x)=x^2-x+3$. If a and h are real numbers, find $\frac{f(a+h)-f(a)}{h}$, if $h \neq 0$. (3.4 #9)

$\frac{(a+h)^2-(a+h)+3-(a^2-a+3)}{h}$

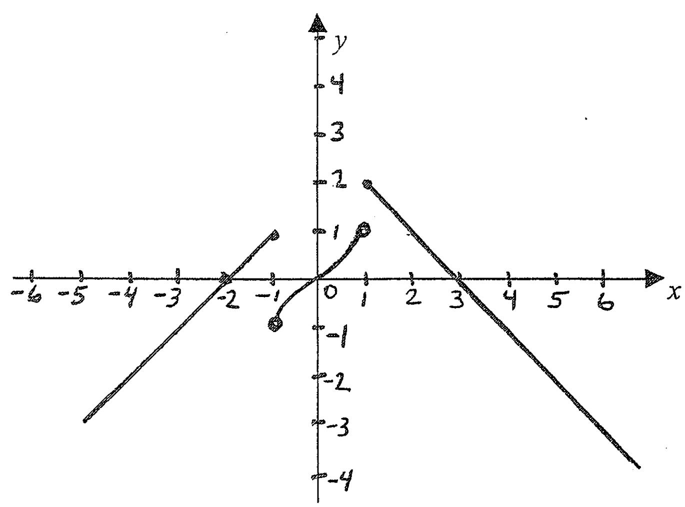
$=\frac{a^2+2ah+h^2-a-h+3-a^2+a-3}{h}$

$=\frac{2ah+h^2-h}{h}=2a+h-1$

15. $=2a+h-1$ (4)

16. Sketch the graph of $f(x)=\begin{cases} x+2 & \text{if } x\leq -1 \\ x^3 & \text{if } -1<x<1 \\ -x+3 & \text{if } x\geq 1 \end{cases}$ (3.5 #51)

(4)



17. Express $f(x)=-3x^2-6x-5$ in the form $f(x)=a(x-h)^2+k$ (3.6 #9)

$y=-3(x^2+2x+1)-5+3$

$f(x)=-3(x+1)^2-2$

17. $f(x)=-3(x+1)^2-2$ (4)

18. Given $f(x)=\dfrac{x-1}{x-2}$ and $g(x)=\dfrac{x-3}{x-4}$. (3.7 #33)

(a) Find $(f \circ g)(x)$.

$f(g(x))=f\left(\frac{x-3}{x-4}\right)$

$= \dfrac{\frac{x-3}{x-4}-1}{\frac{x-3}{x-4}-2} = \dfrac{x-3-1(x-4)}{x-3-2(x-4)} = \dfrac{x-3-x+4}{x-3-2x+8}$

$= \dfrac{1}{5-x}, \ (f \circ g)(x)=\dfrac{1}{5-x}$

18 (a) $(f \circ g)(x)=\dfrac{1}{5-x}$ (4)

(b) Find the domain of $(f \circ g)(x)$.

1) $x \neq 4$

2) $g(x) \neq 2$

$\frac{x-3}{x-4} \neq 2$

$x-3 \neq 2x-8$

$x \neq 5$

$D=\{x \mid x \in \mathbb{R},\ x \neq 4,5\}$

18 (b) $D=\{x \mid x \in \mathbb{R},\ x \neq 4,5\}$ (4)

19. Find the quotient and the remainder if $f(x)=3x^3+2x-4$ is divided $p(x)=2x^2+1$. (4.2 #3)

$$\begin{array}{r} \tfrac{3}{2}x \\ 2x^2+1 \,\overline{)\, 3x^3+0x^2+2x-4} \\ -(3x^3 + \tfrac{3}{2}x) \\ \hline \tfrac{1}{2}x-4 \end{array}$$

$q(x)=\frac{3}{2}x, \ r(x)=\frac{1}{2}x-4$

Check: $f(x)=p(x)q(x)+r(x)$

$(2x^2+1)(\tfrac{3}{2}x)+(\tfrac{1}{2}x-4)=3x^3+\tfrac{3}{2}x+\tfrac{1}{2}x-4$

$=3x^3+2x-4$ ✓

19. $q(x)=\frac{3}{2}x$
$r(x)=\frac{1}{2}x-4$ (4)

20. Determine whether $f(x)=x^2-9$ is one-to-one. (5.1 #7)

Set $f(a)=f(b)$, Show $a=b$ for 1-1.

$a^2-9=b^2-9$

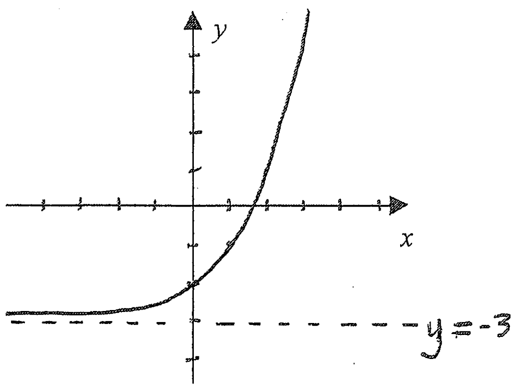
$a^2=b^2$

$a=\pm b$ Therefore $f(x)$ is not 1-1.

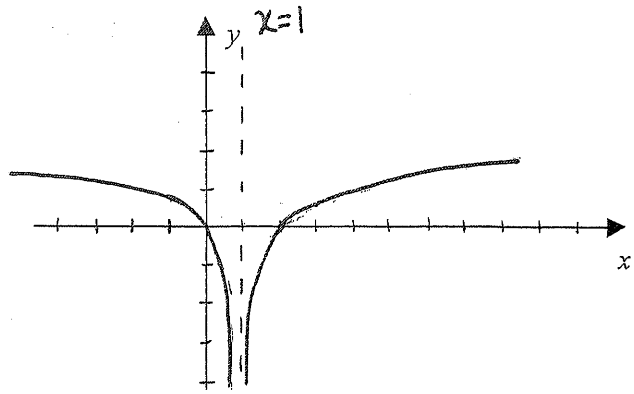
20. $f(x)$ is not one-to-one (4)

21. Sketch the graphs and dash in the asymptotes. (5.2 #11g and 5.4 #40)

a) Graph $f(x) = 2^x - 3$. (2)



b) Graph $f(x) = \ln|x-1|$. (2)



22. ***U.S. population growth*** The population $N(t)$ (in millions) of the United States t years after 1980 may be approximated by the formula $N(t) = 231e^{0.0103t}$. When will the population be twice what it was in 1980? (5.4 #63)

1980 : $t=0$. Find t when $N(t) = 2(231)$ million

$462 = 231e^{0.0103t}$

$2 = e^{0.0103t}$ $t \approx 67.3$ years,

$0.0103t = \ln 2$ which corresponds

$t = \dfrac{\ln 2}{0.0103}$ years $\approx$ to the year 2047.

22. in the year 2047 (4)

23. Solve the equation. (5.5 #23)

$\ln(-4-x) + \ln 3 = \ln(2-x)$

$\ln(-4-x)(3) = \ln(2-x)$

$-12-3x = 2-x$

$-2x = 14$

$x = -7$

23. $x = -7$ (4)

24. Solve the compound interest formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ for t using natural logarithms.

(5.6 #52)

$$\frac{A}{P} = (1+r/n)^{nt}$$
$$\ln(A/P) = \ln(1+r/n)^{nt}$$
$$\ln(A/P) = nt\ln(1+r/n)$$
$$t = \frac{\ln(A/P)}{n\ln(1+r/n)}$$

24. $t = \frac{\ln(A/P)}{n\ln(1+r/n)}$ (4)

Bonus: If \$1000 is deposited in a savings account that pays interest at a rate of 8.25% per year compounded continuously, find the balance after 5 years.

(5.3 #5)

$$A = Pe^{rt}$$
$$A = \$1000e^{0.0825(5)}$$
$$A = \$1000e^{0.4125}$$
$$A \approx \$1510.\underline{59}$$

Bonus: $A \approx \$1510.\underline{59}$ (4)